Unit 6
Algebraic Manipulation

EXERCISE 6.1

Q1. Find the H.C.F. of the following expressions.

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3 \times 13\, x^7y^3z$

$91x^5y^6z^7 = 13 \times 7\, x^5y^6z^7$

$\text{H.C.F.} = 13\, x^5y^3z$

(ii) $102\, xy^2z,\quad 85\, x^2yz$ and $187\, xyz^2$

Solution:

$102\, xy^2z = 2 \times 3 \times 17\, xy^2z$

$85\, x^2yz = 5 \times 17\, x^2yz$

$187\, xyz^2 = 11 \times 17\, xyz^2$

$\text{H.C.F.} = 17xyz$

Q2. Find the H.C.F. of the following expressions by factorization.

(i) $x^2 + 5x + 6,\quad x^2 - 4x - 12$

Solution:

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 3x + 2x + 6 \\ &= x(x+3) + 2(x+3) \\ &= (x+3)(x+2) \end{aligned}$$

$$\begin{aligned} x^2 - 4x - 12 &= x^2 - 6x + 2x - 12 \\ &= x(x-6) + 2(x-6) \\ &= (x-6)(x+2) \end{aligned}$$

$\text{H.C.F.} = x + 2$

(ii) $x^3 - 27,\quad x^2 + 6x - 27,\ 2x^2 - 18$

Solution:

$x^3 - 27 = (x)^3 - (3)^3 = (x-3)(x^2 + 3x + 9)$

$$\begin{aligned} x^2 + 6x - 27 &= x^2 + 9x - 3x - 27 \\ &= x(x+9) - 3(x+9) \\ &= (x+9)(x-3) \end{aligned}$$

$$\begin{aligned} 2x^2 - 18 &= 2(x^2 - 9) = 2(x^2 - 3^2) \\ &= 2(x+3)(x-3) \end{aligned}$$

$H.C.F. = x - 3$

(iii) $x^3 - 2x^2 + x, \quad x^2 + 2x - 3, \quad x^2 + 3x - 4$

Solution:

$$x^3 - 2x^2 + x = x(x^2 - 2x + 1) = x(x-1)^2$$
$$= x(x-1)(x-1)$$
$$x^2 + 2x - 3 = x^2 + 3x - x - 3$$
$$= x(x+3) - 1(x+3)$$
$$= (x+3)(x-1)$$
$$x^2 + 3x - 4 = x^2 + 4x - x - 4$$
$$= x(x+4) - 1(x+4)$$
$$= (x+4)(x-1)$$

$H.C.F. = x - 1$

(iv) $18(x^3 - 9x^2 + 8x), \quad 24(x^2 - 3x + 2)$

Solution:

$$18(x^3 - 9x^2 + 8x) = 18(x^2 - 9x + 8)$$
$$= 18x(x^2 - 8x - x + 8)$$
$$= 18x[x(x-8) - 1(x-8)]$$
$$= 2 \times 3^2 \times (x-2)(x-1)$$

$H.C.F. = 2 \times 3(x-1) = 6(x-1)$

(v) $36(3x^4 + 5x^3 - 2x^2), \quad 54(27x^4 - x)$

Solution:

$$36(3x^4 + 5x^3 - 2x^2) = 4 \times 9x^2(3x^2 + 5x - 2)$$
$$= 4 \times 9x^2(3x^2 + 6x - x - 2)$$
$$= 2^2.3^2\, x^2[3x(x+2) - 1(x+2)]$$
$$= 2^2.3^2\, x^2\,(x+2)(3x-1)$$
$$54(27x^4 - 2) = 2 \times 27x(27x^3 - 1)$$
$$= 2 \times 3^3x[(3x)^3 - (1)^3]$$
$$= 2 \times 3^3x(3x-1)(9x^2 + 3x + 1)$$

$H.C.F. = 2 \times 3^2x(3x - 1) \quad = 18x(3x-1)$

Q3. Find the H.C.F. of the following by division method.

(i) $x^3 + 3x^2 - 16x + 12, \quad x^3 + x^2 - 10x + 8$

Solution:

$$\begin{array}{r|l} & \quad 1 \\ x^3 + x^2 - 10x + 8 & x^3 + 3x^2 - 16x + 12 \\ & \underline{\pm x^3 \pm x^2 \mp 10x \pm 8} \\ & 2x^2 - 6x + 4 \\ & 2(x^2 - 3x + 2) \end{array}$$

By Ignoring 2

$$
\begin{array}{r|l}
 & \qquad\quad x+4 \\
x^2-3x+2 & x^3+x^2-10x+8 \\
 & \underline{\pm x^3 \mp 3x^2 \pm 2x} \\
 & 4x^2-12x+8 \\
 & \underline{\pm 4x^2 \mp 12x \pm 8} \\
 & \qquad 0
\end{array}
$$

H. C. F. $= \boldsymbol{x^2-3x+2}$

(ii) $\boldsymbol{x^4+x^3-2x^2-x-3,\quad 5x^3+3x^2-17x+6}$

Solution:

$$
\begin{array}{r|l}
 & \qquad\qquad x+2 \\
5x^3+3x^2-17x+6 & x^4+x^3-2x^2-x-3 \\
 & \underline{\times 5} \\
 & 5x^4+5x^3-10x^2-5x-15 \\
 & \underline{\pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x} \\
 & 2x^3+7x^2-x-15 \\
 & \underline{\times 5} \\
 & 10x^3+35x^2-5x-75 \\
 & \underline{\pm 10x^3 \pm 6x^2 \mp 34x \pm 12} \\
 & \qquad 29x^2+29x-87 \\
 & \qquad 29(x^2+x-3)
\end{array}
$$

By Ignoring 29

$$
\begin{array}{r|l}
 & \qquad 5x-2 \\
x^2+x-3 & 5x^3+3x^2-17x+6 \\
 & \underline{\pm 5x^3 \pm 5x^2 \mp 15x} \\
 & \quad -2x^2-2x+6 \\
 & \quad \underline{\mp 4x^2 \mp 2x \pm 6} \\
 & \qquad\quad 0
\end{array}
$$

H. C. F. $= \boldsymbol{x^2+x-3}$

(iii) $\boldsymbol{2x^5-4x^4-6x,\quad x^5+x^4-3x^3-3x^2}$

Solution:

$2x^5-4x^4-6x = 2x(x^4-2x^3-3)$

$x^5+x^4-3x^3-3x^2 = x^2(x^3+x^2-3x-3)$

In this case H.C.F. of $2x$ and x^2 is x

Now we find H.C.F. of x^4-2x^3 and x^3+x^2-3x-3

$$
\begin{array}{r|l}
 & x-3 \\ \hline
x^3+x^2-3x-3 & x^4-2x^3-x^2+x-3 \\
 & \pm x^4 \pm x^3 \mp 3x^3 \mp 3x \\ \hline
 & -3x^3+3x^2+3x-3 \\
 & \mp 3x^3 \mp 3x^2 \pm 3x \pm 3 \\ \hline
 & 6x^2-6x-12 \\
 & 6(x^2-x-2)
\end{array}
$$

By Ignoring 6

$$
\begin{array}{r|l}
 & x+2 \\ \hline
x^2-x-2 & x^3+x^2-3x-3 \\
 & \pm x^3 \mp x^2 \mp 2x \\ \hline
 & 2x^2-x-3 \\
 & \pm 2x^2 \mp 2x \mp 4 \\ \hline
 & x+1
\end{array}
$$

Then

$$
\begin{array}{r|l}
 & x-2 \\ \hline
x+1 & x^2-x-2 \\
 & \pm x^2 \pm x \\ \hline
 & -2x-2 \\
 & \mp 2x \mp 2 \\ \hline
 & 0
\end{array}
$$

H. C. F. = x + 1

Hence the H.C.F. of the given expression is

$$x \times (x+1) = x^2 + x$$

Q4. Find the L.C.M. of the following expressions.

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution:

$$39x^7y^3z = 3 \times 13\ x^7y^3z$$
$$91x^5y^6z^7 = 13 \times 7\ x^5y^6z^7$$
$$\text{L.C.M.} = 3 \times 7 \times 13\ x^7y^6z^7$$
$$= 273\ x^7y^6z^7$$

(ii) $102\ xy^2z$, $85\ x^2yz$ and $187\ xyz^2$

Solution:

$$102\ xy^2z = 2 \times 3 \times 17\ xy^2z$$
$$85\ x^2yz = 5 \times 17\ x^2yz$$
$$187\ xyz^2 = 11 \times 17\ xyz^2$$
$$\text{L.C.M.} = 2 \times 3 \times 5 \times 11 \times 17x^2y^2z^2$$
$$= 5610\ x^2y^2z^2$$

Q5. Find the L.C.M. of the following expressions by factorization.

(i) $x^2 + 25x + 100$ and $x^2 - x - 20$

Solution:

$$x^2 + 25x + 100 = x^2 - 20x - 5x + 100$$
$$= x(x - 20) - 5(x - 20)$$
$$= (x - 20)(x - 5)$$
$$x^2 - x - 20 = x^2 - 5x + 4x - 20$$
$$= x(x - 5) + 4(x - 5)$$
$$= (x - 5)(x + 4)$$
$$\text{L.C.M.} = (x - 5)(x - 20)(x + 4)$$

(ii) $x^2 + 4x + 4,\quad x^2 - 4,\quad 2x^2 + x - 6$

Solution:

$$x^2 + 4x + 4 = (x + 2)^2$$
$$x^2 - 4 = (x + 2)(x - 2)$$
$$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6$$
$$= 2x(x + 2) - 3(x + 2)$$
$$= (x + 2)(2x - 3)$$
$$\text{L.C.M.} = (x + 2)^2(x - 2)(2x - 3)$$

(iii) $2(x^4 - y^4), 3(x^3 + 2x^2y - xy^2 - 2y^3)$

Solution:

$$2(x^4 - y^4) = 2(x^2 - y^2)(x^2 + y^2)$$
$$= 2(x - y)(x + y)$$
$$= 2(x^2 - y^2)(x^2 + y^2)(x^2 + y^2)$$
$$3(x^3 + 2x^2y - xy^2 - 2y^3) = 3[x^2(x + 2y) - y^2(x + 2y)]$$
$$= 3(x + 2y)(x^2 + y^2)$$
$$= 3(x + y)(x - y)(x + 2y)$$
$$\text{L.C.M.} = 2\,.\,3(x - y)(x + y)(x^2 + y^2)(x + 2y)$$
$$= 6(x^2 - y^2)(x^2 + y^2)(x + 2y)$$
$$= 6(x^4 - y^4)(x + 2y)$$

(iv) $4(x^4 - 1),\quad 6(x^3 - x^2 - x + 1)$

Solution:

$$4(x^4 - 1) = 4(x^2 - 1)(x^2 + 1)$$
$$= 2^2(x - 1)(x + 1)(x^2 + 1)$$
$$6(x^3 - x^2 - x + 1) = 6[x^2(x - 1) - 1(x - 1)]$$
$$= 2\,.\,3(x - 1)(x^2 - 1)$$
$$= 2\,.\,3(x - 1)(x - 1)(x + 1)$$
$$= 2\,.\,3(x - 1)^2(x + 1)$$

$$
\begin{aligned}
L.C.M. &= 2^2 .3(x-1)^2(x+1)(x^2+1)\\
&= 12(x-1)^2(x+1)(x^2+1)\\
&= 12(x-1)(x-1)(x+1)(x^2+1)\\
&= 12(x-1)(x^2-1)(x^2+1)\\
&= 12(x-1)(x^4-1)
\end{aligned}
$$

Q6. For what value of k is $(x+4)$ the H.C.F of $(x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

Let $P(x) = x^2+x-(2k+2)$

And $q(x) = 2x^2+kx-12$

As $x+4$ is H.C.F. of p(x) and q(x). So p(x)is exactly divisible by $x+4$ and thus $p(-4) = 0$

i.e. $(-4)^2+(-4)-(2k+2)=0$

$= \quad 16-4-2k-2=0$

$\Rightarrow \quad 10-2k=0 \quad \Rightarrow \quad 2k=10$

$k=5$

Q7. If $(x+3)(x-2)$ is the H.C.F. of $p(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2+7x-1)$, find k and l.

Solution:

$p(x)=(x+3)(2x^2-3x+k)$

$q(x)=(x-2)(3x^2+7x-1)$

H.C.F. of $p(x)$ and $q(x)=(x+3)(x-2)$

$(x+3)(x-2)$ is a factor of

$(x+3)(2x^2-3x+k)$

Hence $x-2$ is a factor of $2x^2-3x+k$

$\therefore \quad 2(2)^2-3(2)+k=0$

$\Rightarrow \quad 8-6+k=0 \quad \Rightarrow \quad 2+k=0$

$\Rightarrow \quad k=-2$

Similarly $(x+3)(x-2)$is a factor of $(x-2)(3x^2+7x-l)$

$\Rightarrow \quad x+3$ is a factor of $(3x^2+7x-l)$

$3(-3)^2+7(-3)-l=0$

$27-21-l=0$

$6-l=0$

$\Rightarrow \quad l=6$

Q8. The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $p(x)=x^3+x^2+x+1$, find $q(x)$.

Solution:

$$\text{L.C.M.} = 2(x^4 - 1)$$
$$\text{H.C.F.} = (x+1)(x^2+1)$$
$$p(x) = x^3 + x^2 + x + 1$$
$$q(x) = \frac{(L.C.M.)\times(H.C.F.)}{p(x)}$$
$$= \frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$
$$= \frac{2(x^4-1)x^3+x+x^2+1}{x^3+x^2+x+1}$$
$$= \frac{2(x^4-1)(x^3+x^2+x+1)}{x^3+x^2+x+1}$$
$$= 2(x^4-1)$$

Q9. **Let $p(x) = 10(x^2-9)(x^2-3x+2)$ and $q(x) = 10x(x+3)(x-1)^2$. If the H.C.F. of $p(x), q(x)$ is $10(x+3)(x-1)$, find their L.C.M.**

Solution:

$$p(x) = 10(x^2-9)(x^2-3x+2)$$
$$q(x) = 10x(x+3)(x-1)^2$$
$$H.C.F. = 10(x+3)(x-1)$$
$$\text{L.C.M.} = \frac{p(x)\times q(x)}{H.C.F.}$$
$$= \frac{10(x^2-9)(x^2-3x+2)10x(x+3)(x-1)^2}{10(x+3)(x-1)}$$
$$= 10(x^2-9)(x^2-3x+2)\,.\,x(x-1)$$
$$= 10(x^2-9)[(x^2-2x-x+2)\,x.(x-1)]$$
$$= 10(x^2-9)[x(x-2)-1(x-2].x.(x-1)$$
$$= 10(x^2-9)(x-2)(x-1)\,.\,x\,.\,(x-1)$$
$$= 10(x^2-9)(x-2)\,.\,x\,.\,(x-1)^2$$
$$= 10x(x-2)(x-1)^2(x^2-9)$$

Q10. **Let the product of L.C.M. and H.C.F. two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.**

Solution:

$$(L.C.M.)(H.C.F.) = (x+3)^2(x-2)(x+5)$$
$$p(x) = (x+3)(x-2)$$
$$q(x) = x^2 + kx + 15$$
$$p(x)\,.\,q(x) = (L.C.M.)\times(H.C.F.)$$

$$(x+3)(x-2)(x^2+kx+15) = (x+3)^2(x-2)(x+5)$$

$\Rightarrow$ $$x^2+kx+15 = (x+3)(x+5)$$

$$x^2+kx+15 = x^2+8x+15$$

$\Rightarrow$ $$k = 8$$

Q11. Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

Solution:

Required number of children = H.C.F. of 128 and 176.

$$\begin{array}{rrrrr}
 & 1 & & & \\
128 \,\big)\, & 176 & & & \\
 & 128 & 2 & & \\ \hline
 & 48 \,\big)\, & 128 & & \\
 & & 96 & 1 & \\ \hline
 & & 32 \,\big)\, & 48 & \\
 & & & 32 & 2 \\ \hline
 & & & 16 \,\big)\, & 32 \\
 & & & & 32 \\ \hline
 & & & & 0
\end{array}$$

H.C.F. = 16

Hence the highest number of children = 16.

EXERCISE 6.2

Simplify each of the following as a rational expression.

Q1. $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

Solution:

$$=\frac{x^2-3x+2x-6}{x^2-3^2}+\frac{x^2+6x-4x-24}{x^2-4x+3x-12}$$

$$=\frac{x(x-3)+2(x-3)}{(x+3)(x-3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$=\frac{(x-3)(x+2)}{(x+3)(x-3)}+\frac{(x+6)(x-4)}{(x-4)(x+3)}$$

$$=\frac{x+2}{x+3}+\frac{x+6}{x+3}=\frac{x+2+x+6}{x+3}=\frac{2x+8}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q2. $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

Solution:

$$=\frac{(x+1)^2(x^2+1)-(x-1)^2(x^2+1)-4x(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{(x^2+2x+1)(x^2+1)-(x^2-2x+1)(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+x^2+2x^3+2x+1-(x^4+x^2-2x^3-2x+x^2+1)-(4x^3-4x)}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+2x^3+2x^2+2x+1-x^4+2x^3-2x^2+2x-1-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{4x^3+4x-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}\qquad=\frac{12x}{x^4-1}$$

Q3. $\dfrac{1}{x^2-8x+15}+\dfrac{1}{x^2-4x+3}-\dfrac{2}{x^2-6x+5}$

Solution:

$$=\frac{1}{x^2-5x-3x+15}+\frac{1}{x^2-3x-x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-5)-3(x-5)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-5)(x-3)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{x-1+x-5-2(x-3)}{(x-5)(x-3)(x-1)}$$

$$=\frac{2x-6-2x+6}{(x-5)(x-3)(x-1)}=\frac{0}{(x-5)(x-3)(x-1)}$$

$$=0$$

Q4. $\dfrac{(x+2)(x+3)}{x^2-9}+\dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$=\frac{(x+2)(x+3)}{x^2-3^2}+\frac{(x+2).2(x^2-16)}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)(x+4)(x-4)}{(x-4)[x(x-3)+2(x-3)]}$$

$$=\frac{x+2}{x-3}+\frac{2(x+2)(x+4)}{(x-3)(x+2)}$$

$$=\frac{x+2}{x-3}+\frac{2(x+4)}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}\qquad=\frac{3x+10}{x-3}$$

Q5. $\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

Solution:

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-3^2}$$
$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$
$$=\frac{x+3}{(x+3)(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$
$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$
$$=\frac{2(2x-3)+2x+3-(4x)2}{2(2x+3)(2x-3)}$$
$$=\frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)}$$
$$=\frac{-2x-3}{2(2x+3)(2x-3)} \qquad =\frac{-(2x+3)}{2(2x+3)(2x-3)}$$
$$=\frac{-1}{2(2x-3)} \qquad =\frac{1}{2(3-2x)}$$

Q6. $A-\dfrac{1}{A}$, **where** $A=\dfrac{a+1}{a-1}$

Solution:

$$A-\frac{1}{A}=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$
$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$
$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$
$$=\frac{a^2+2a+1-a^2+2a-1}{a^2-1}=\frac{4a}{a^2-1}$$

Q7. $\left[\dfrac{x-1}{x-2}+\dfrac{2}{2-x}\right]-\left[\dfrac{x+1}{x+2}+\dfrac{4}{4-x^2}\right]$

Solution:

$$=\left[\frac{x-1}{x-2}+\frac{2}{-(x-2)}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-(x^2-4)}\right]$$
$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$$
$$=\frac{x-1-2}{x-2}-\frac{(x+1)(x-2)-4}{x^2-4}$$
$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{x^2-4}$$
$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{x^2-4}$$
$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{x^2-2^2}$$

$$= \frac{x-3}{x-2} - \frac{x(x-3)+2(x-3)}{(x+2)(x-2)}$$
$$= \frac{x-3}{x-2} - \frac{(x-3)(x+2)}{(x+2)(x-2)}$$
$$= \frac{x-3}{x-2} - \frac{x-3}{x-2}$$
$$= 0$$

Q8. What rational expression should be subtracted from $\frac{2x^2+2x-7}{x^2+x-6}$ to get $\frac{x-1}{x-2}$?

Solution:

Required expression

$$= \frac{2x^2+2x-7}{x^2+x-6} - \frac{x-1}{x-2}$$
$$= \frac{2x^2+2x-7}{x^2+3x-2x-6} - \frac{x-1}{x-2}$$
$$= \frac{2x^2+2x-7}{(x+3)(x-2)} - \frac{x-1}{x-2}$$
$$= \frac{2x^2+2x-7-(x+3)(x-1)}{(x+3)(x-2)}$$
$$= \frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$
$$= \frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$
$$= \frac{x^2-4}{(x+3)(x-2)} = \frac{(x+2)(x-2)}{(x+3)(x-2)}$$
$$= \frac{x+2}{x+3}$$

Perform the indicated operations and simplify to the lowest forms.

Q9. $\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$

Solution:

$$= \frac{x^2+3x-2x-6}{x^2-3x+2x-6} \times \frac{x^2-2^2}{x^2-3^2}$$
$$= \frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$
$$= \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$
$$= \frac{(x+3)(x-2)}{(x-3)(x+2)}$$
$$= \frac{(x-2)^2}{(x-3)^2}$$

Q10. $\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$

Solution:

$$= \frac{x^3-2^3}{x^2-2^2} \times \frac{x^2+4x+2x+8}{(x-1)^2}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} \times \frac{x(x+4)+2(x+4)}{(x-1)(x-1)}$$

$$= \frac{x^2+2x+4}{x+2} \times \frac{(x+4)(x+2)}{(x-1)(x-1)}$$

$$= \frac{(x+4)(x^2+2x+4)}{(x-1)^2}$$

Q11. $\frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x^2-2x}$

Solution:

$$= \frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x^2-2x}$$

$$= \frac{x(x^3-8)}{2x^2+6x-x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{2x(x+3)-1(x+3)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{(x+3)(2x-1)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= 1$$

Q12. $\frac{2y^2+7y-4}{3y^2-13y+4} \div \frac{4y^2-1}{6y^2+y-1}$

Solution:

$$= \frac{2y^2+7y-4}{3y^2-13y+4} \times \frac{6y^2+y-1}{4y^2-1}$$

$$= \frac{2y^2+8y-y-4}{3y^2-12y-y+4} \times \frac{6y^2+3y-2y-1}{(2y)^2-1^2}$$

$$= \frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)} \times \frac{3y(2y+1)-1(2y+1)}{(2y+1)(2y-1)}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \times \frac{(2y+1)(3y-1)}{(2y+1)(2y-1)}$$

$$= \frac{y+4}{y-4}$$

Q13. $\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

Solution:

$$= \frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)} \div \frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}$$

$$= \frac{x^4+2x^2y^2+y^4-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-(x^2-2xy+y^2)}{(x-y)(x+y)}$$

$$= \frac{x^4+2x^2y^2+y^4-x^4+2x^2y^2-y^4}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x-y)(x+y)}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \div \frac{4xy}{(x^2-y^2)}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy}$$

$$= \frac{xy}{x^2+y^2}$$

EXERCISE 6.3

Q1. Use factorization to find the square root of the following expressions.

(i) $4x^2 - 12xy + 9y^2$

Solution:

$$= (2x)^2 - 2(2x)(3y) + (3y)^2$$

$$= (2x - 3y)^2$$

$$= \sqrt{(2x-3y)^2}$$

Required square root is $\pm(2x - 3y)$

(ii) $x^2 - 1 + \frac{1}{4x^2} \quad (x \neq 0)$

Solution:

$$= (x)^2 - 2(x)\left(\frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^2$$

$$= \left(x - \frac{1}{2x}\right)^2$$

Required square root is $\pm\left(x - \frac{1}{2x}\right)$

(iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$

Solution:

$$= \left(\frac{1}{4}x\right)^2 - 2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right) + \left(\frac{1}{6}y\right)^2$$

$$= \left(\frac{1}{4}x - \frac{1}{6}y\right)^2$$

Required square root is $\frac{1}{4}x - \frac{1}{6}y$

or $\pm\left(\frac{x}{4} - \frac{y}{6}\right)$

(iv) $4(a+b)^2 - 12(a^2-b^2) + 9(a-b)^2$

Solution:

$$= [2(a+b)]^2 - 2[2(a+b)][3(a-b)] + [3(a-b)]^2$$
$$= [2(a+b) - 3(a-b)]^2$$
$$= (2a+2b-3a+3b)^2$$
$$= (5b-a)^2$$

Required square root is $\pm(5b-a)$

(v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

$$= \frac{(2x^3)^3-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2-2(3x^2)(4y^2)+(4y^2)^2}$$
$$= \frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2} = \left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

$\therefore$ Required square root is $\pm\left(\dfrac{2x^3-3y^3}{3x^2+4y^2}\right)$

(vi) $\left(x+\frac{1}{x}\right)^2 - 4\left(x-\frac{1}{x}\right) \qquad (x \neq 0)$

Solution:

$$= x^2+2+\frac{1}{x^2} - 4\left(x-\frac{1}{x}\right)$$
$$= x^2-2+\frac{1}{x^2} - 4\left(x-\frac{1}{x}\right) + 4$$
$$= \left(x-\frac{1}{x}\right)^2 - 4\left(x-\frac{1}{x}\right) + 4$$
$$= \left(x-\frac{1}{x}\right)^2 - 2\left(x-\frac{1}{x}\right).2 + (2)^2$$
$$= \left[\left(x-\frac{1}{x}\right) - 2\right]^2$$

$\therefore$ Required square root is $\pm\left[\left(x-\frac{1}{x}\right)-2\right]$

(vii) $\left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x+\frac{1}{x}\right)^2 + 12 \qquad (x \neq 0)$

Solution:

$$= x^4+2+\frac{1}{x^4} - 4\left(x^2+2+\frac{1}{x^2}\right) + 12$$
$$= \left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x^2+\frac{1}{x^2}\right) - 8 + 12$$
$$= \left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x^2+\frac{1}{x^2}\right) + 4$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2\left(x^2 + \frac{1}{x^2}\right).2 + (2)^2$$

$$= \left[\left(x^2 + \frac{1}{x^2}\right) - 2\right]^2$$

Required square root is $\pm\left[\left(x^2 + \frac{1}{x^2}\right) - 2\right]$

(viii) $(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)$

Solution:

$$= (x^2 + 2x + x + 2)(x^2 + 3x + x + 3)(x^2 + 3x + 2x + 6)$$

$$= [x(x + 2) + 1(x + 2)][x(x + 3) + 1(x + 3)]$$
$$[x(x + 3) + 2(x + 3)]$$

$$= (x + 2)(x + 1)(x + 3)(x + 1)(x + 3)(x + 2)$$

$$= (x + 1)^2(x + 2)^2(x + 3)^2$$

$$= [(x + 1)(x + 2)(x + 3)]^2$$

$\therefore$ Required square root is $\pm[(x + 1)(x + 2)(x + 3)]$

(ix) $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

Solution:

$$= (x^2 + 7x + x + 7)(2x^2 - 3x + 2x - 3)$$
$$(2x^2 + 14x - 3x - 21)$$

$$= [x(x + 7) + 1(x + 7)][x(2x - 3) + 1(2x - 3)]$$
$$[2x(x + 7) - 3(x + 7)]$$

$$= (x + 7)(x + 1)(2x - 3)(x + 1)(x + 7)(2x - 3)$$

$$= (x + 7)^2(x + 1)^2(2x - 3)^2$$

$$= [(x + 7)(x + 1)(2x - 3)]^2$$

So the required square root is $\pm[(x + 1)(x + 7)(2x - 3)]$

Q2. Use division method to find the square root of the following expressions.

(i) $4x^2 + 12xy + 9y^2 + 16x + 24y + 16$

Solution:

$$\begin{array}{r|l}
 & \quad 2x + 3y + 4 \\ \hline
2x & 4x^2 + 12xy + 9y^2 + 16x + 24y + 16 \\
 & \pm 4x^2 \\ \hline
4x + 3y & 12xy + 16x + 9y^2 + 24y + 16 \\
 & \pm 12xy \qquad \pm 9y^2 \\ \hline
4x + 6y + 4 & 16x \qquad +24y + 16 \\
 & \pm 16x \qquad \pm 24y \pm 16 \\ \hline
 & \qquad 0
\end{array}$$

$\therefore$ The square root is $\pm(2x + 3y + 4)$

(ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$

Solution:

$$
\begin{array}{r|l}
 & \qquad\qquad x^2 - 5x + 6 \\
\hline
x^2 & x^4 - 10x^3 + 37x^2 - 60x + 36 \\
 & \pm x^4 \\
\hline
2x^2 - 5x & \quad -10x^3 + 37x^2 \\
 & \quad \mp 10x^3 \pm 25x^2 \\
\hline
2x^2 - 10x + 6 & \qquad\qquad 12x^2 - 60x + 36 \\
 & \qquad\qquad \pm 12x^2 \mp 60x \pm 36 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

$\therefore$ The square root is $\pm(x^2 - 5x + 6)$.

(iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$

Solution:

$$
\begin{array}{r|l}
 & \qquad\qquad 3x^2 - x + 1 \\
\hline
3x^2 & 9x^4 + 6x^3 + 7x^2 - 2x + 1 \\
 & \pm 9x^4 \\
\hline
6x^2 - x & \quad -6x^3 + 7x^2 \\
 & \quad \mp 6x^3 \pm x^2 \\
\hline
6x^2 - 2x + 1 & \qquad\qquad 6x^2 - 2x + 1 \\
 & \qquad\qquad \pm 6x^2 \mp 2x \pm 1 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

$\therefore$ The square root is $\pm(3x^2 - x + 1)$.

(iv) $4 + 25x^2 - 12x - 24x^3 + 16x^4$

Solution:

$$
\begin{array}{r|l}
 & \qquad\qquad 4x^2 - 3x + 2 \\
\hline
4x^2 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\
 & \pm 16x^4 \\
\hline
8x^2 - 3x & \quad -24x^3 + 25x^2 \\
 & \quad \mp 24x^3 \pm 9x^2 \\
\hline
8x^2 - 6x + 2 & \qquad\qquad 16x^2 - 12x + 4 \\
 & \qquad\qquad \pm 16x^2 \mp 12x \pm 4 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

The square root is $\pm(4x^2 - 3x + 2)$.

(v) $\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{x}{y} + \frac{x^2}{y^2}$ $\quad (x \neq 0)(y \neq 0)$

Solution:

$$
\begin{array}{l|l}
 & \frac{x}{y} - 5 + \frac{y}{x} \\
\hline
\frac{x}{y} & \frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{x}{y} + \frac{x^2}{y^2} \\
 & \pm\frac{x^2}{y^2} \\
\hline
2\frac{x}{y} - 5 & -10\frac{x}{y} + 27 \\
 & \mp 10\frac{x}{y} \pm 25 \\
\hline
2\frac{x}{y} - 10 + \frac{y}{x} & 2 - 10\frac{x}{y} + \frac{y^2}{x^2} \\
 & \pm 2 \mp 10\frac{x}{y} \pm \frac{y^2}{x^2} \\
\hline
 & 0
\end{array}
$$

So the square root is $\pm\left(\frac{x}{y} - 5 + \frac{y}{x}\right)$.

Q3. Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

Solution:

$$
\begin{array}{l|l}
 & 2x^2 - 3x + 7 \\
\hline
2x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + k \\
 & \pm 4x^4 \\
\hline
84x^2 - 3x & -12x^3 + 37x^2 \\
 & \mp 12x^3 \pm 9x^2 \\
\hline
4x^2 - 6x + 7 & 28x^2 - 42x + k \\
 & \pm 28x^2 \mp 42x \pm 49 \\
\hline
 & k - 49
\end{array}
$$

The given expression will be perfect square when remainder = 0

if $\quad k - 49 = 0$

i.e. $\quad k = 49$

(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

Solution:

$$\begin{array}{l|l}
 & x^2 - 2x + 3 \\ \hline
x^2 & x^4 - 4x^3 + 10x^2 - kx + 9 \\
 & \pm x^4 \\ \hline
2x^2 - 2x & -4x^3 + 10x^2 - kx + 9 \\
 & \mp 4x^3 \pm 4x^2 \\ \hline
2x^2 - 4x + 3 & 6x^2 - kx + 9 \\
 & \pm 6x^2 \mp 12x \pm 9 \\ \hline
 & -kx + 12x
\end{array}$$

The given expression will be perfect square when remainder = 0

if $-kx + 12x = 0$

$\Rightarrow$ $-k + 12 = 0$

$\Rightarrow$ $-k = -12$

i.e. $k = 12$

Q4. Find the value of l and m for which the following expressions will become a perfect squares.

(i) $x^4 + 4x^3 + 16x^2 - lx + m$

Solution:

$$\begin{array}{l|l}
 & x^2 + 2x + 6 \\ \hline
x^2 & x^4 + 4x^3 + 16x^2 + lx + m \\
 & \pm x^4 \\ \hline
2x^2 + 2x & 4x^3 + 16x^2 + lx + m \\
 & \mp 4x^3 \pm 4x^2 \\ \hline
2x^2 + 4x + 6 & 12x^2 + lx + m \\
 & \pm 12x^2 \pm 24x \pm 36 \\ \hline
 & (l - 24)x - (m - 36)
\end{array}$$

The given expression will be perfect square when remainder = 0

if $l - 24 = 0$

and $m - 36 = 0$

$\therefore$ $l = 24,$ $m = 36.$

(ii) $\quad 49x^4 - 70x^3 + 109x^2 + lx - m$

Solution:

$$
\begin{array}{r|l}
 & \quad 7x^2 - 5x + 6 \\
\hline
7x^2 & 49x^4 - 70x^3 + 109x^2 + lx - m \\
 & \pm 49x^4 \\
\hline
14x^2 - 5x & \quad -70x^3 + 109x^2 \\
 & \quad \mp 70x^3 \pm 25x^2 \\
\hline
14x^2 - 10x + 6 & \qquad 84x^2 + lx - m \\
 & \qquad \pm 84x^2 \mp 60x \pm 36 \\
\hline
 & \qquad (l + 60)x - m - 36
\end{array}
$$

The given expression will be perfect square

if $\quad l + 60 = 0 \quad$ and $\quad -m - 36 = 0$

i.e. $\quad l = -60 \quad ; \quad m = -36$

Q5. **To make the expression $9x^4 - 12x^3 + 22x^2 - 13x + 12$, a perfect square**

(i) **what should be added to it?**

(ii) **what should be subtracted from it?**

(iii) **what should be the value of x?**

Solution:

$$
\begin{array}{r|l}
 & \quad 3x^2 - 2x + 3 \\
\hline
3x^2 & 9x^4 - 12x^3 + 22x^2 - 13x + 12 \\
 & \pm 9x^4 \\
\hline
6x^2 - 2x & \quad -12x^3 + 22x^2 \\
 & \quad \mp 12x^3 \pm 4x^2 \\
\hline
6x^2 - 4x + 3 & \qquad 18x^2 - 13x + 12 \\
 & \qquad \pm 18x^2 \mp 60x \pm 9 \\
\hline
 & \qquad\qquad -x + 3
\end{array}
$$

(i) To make the expression a perfect square we should add $x - 3$.

(ii) To make the expression a perfect square we should subtract $-x + 3$.

(iii) To make the expression a perfect square remainder = 0

$$-x - 3 = 0$$

or $\quad x = 3$

REVIEW EXERCISE 6

Q1. Choose the correct answer.

(i) H.C.F. of $p^3q - pq^3$ and $p^5q^2 - p^2q^5$ is......

(a) $pq(p^2 - q^2)$ (b) $pq(p - q)$
(c) $p^2q^2(p - q)$ (d) $pq(p^3 - q^3)$

(ii) H.C.F. of $5x^2y^2$ and $20x^3y^3$ is......

(a) $5x^2y^2$ (b) $20x^3y^3$
(c) $100x^5y^5$ (d) $5xy$

(iii) H.C.F. of $x - 2$ and $x^2 + x - 6$ is.......

(a) $x^2 + x - 6$ (b) $x + 3$
(c) $x - 2$ (d) $x + 6$

(iv) H.C.F. of $a^3 + b^3$ and $a^2 - ab + b^2$ is......

(a) $a + b$ (b) $a^2 - ab + b^2$
(c) $(a - b)^2$ (d) $a^2 + b^2$

(v) H.C.F. of $x^2 - 5x + 6$ and $x^2 - x - 6$ is.......

(a) $x - 3$ (b) $x + 2$
(c) $x^2 - 4$ (d) $x - 2$

(vi) H.C.F. of $a^2 - b^2$ and $a^3 - b^3$ is......

(a) $a - b$ (b) $a + b$
(c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$

(vii) H.C.F. of $x^2 + 3x + 2$, $x^2 + 4x + 3$ and $x^2 + 5x + 4$ is......

(a) $x + 1$ (b) $(x + 1)(x + 2)$
(c) $x + 3$ (d) $(x + 4)(x + 1)$

(viii) L.C.M. of $15x^2$, $45xy$ and $30xyz$ is ....

(a) $90xyz$ (b) $90x^2yz$
(c) $15xyz$ (d) $15x^2yz$

(ix) L.C.M. of $a^2 + b^2$ and $a^4 - b^4$ is ......

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a^4 - b^4$ (d) $a - b$

(x) The product of algebraic expressions is equal to the ...... of their H.C.F. and L.C.M.

(a) Sum (b) Difference
(c) Product (d) Quotient

(xi) Simplify $\frac{a}{9a^2-b^2} + \frac{1}{3a-b} = \cdots$

(a) $\frac{4a}{9a^2-b^2}$ (b) $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ (d) $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18} \times \frac{a+3}{a-2} = \cdots$

(a) $\frac{a+7}{a-6}$ (b) $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ (d) $\frac{a-2}{a+3}$

(xiii) **Simplify** $\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2} = \cdots$

(a) $\frac{1}{a+b}$ (b) $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ (d) $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y} - 1\right) \div \left(1 - \frac{x}{x+y}\right) = \cdots$

(a) $\frac{x}{x+y}$ (b) $\frac{y}{x+y}$

(c) $\frac{y}{x}$ (d) $\frac{x}{y}$

(xv) **The square root of $a^2 - 2a + 1$ is......**

(a) $\pm(a+1)$ (b) $\pm(a-1)$

(c) $a-1$ (d) $a+1$

(xvi) **What should be added to complete the square of $x^4 + 64$**

(a) $8x^2$ (b) $-8x^2$

(c) $16x^2$ (d) $4x^2$

(xvii) **The square root of $x^4 + \frac{1}{x^4} + 2$ is......**

(a) $\pm\left(x+\frac{1}{x}\right)$ (b) $\left(x^2+\frac{1}{x^2}\right)$

(c) $\pm\left(x-\frac{1}{x}\right)$ (d) $\pm\left(x^2-\frac{1}{x^2}\right)$

Answers:

(i) b	**(ii) a**	**(iii) c**	**(iv) b**	**(v) a**
(vi) a	**(vii) a**	**(viii) b**	**(ix) c**	**(x) c**
(xi) c	**(xii) a**	**(xiii) a**	**(xiv) d**	**(xv) b**
(xvi) c	**(xvii) b**			

Q2. **Find the H.C.F. of the following by factorization.**

$8x^4 - 128$, $12x^3 - 96$

Solution:

$$8x^4 - 128 = 8(x^4 - 16) = 8(x^4 - 4^2)$$
$$= 8(x^2 - 2^2)(x^2 + 2^2)$$

$$\begin{aligned}
\text{H.C.F.} &= 2^3(x-2)(x+2)(x^2+4) \\
12x^3-96 &= 12(x^3-8) = 2^2\times 3(x^3-2^3) \\
&= 2^2\times 3(x-2)(x^2+2x+4) \\
\text{H.C.F.} &= 2^2(x-2) \\
&= 4(x-2)
\end{aligned}$$

Q3. Find the H.C.F. of the following by division method.

$y^3+3y^2-3y-9,\ y^3+3y^2-8y-24$

Solution:

$$\begin{array}{r|c}
 & 1 \\ \hline
y^3+3y^2-3y-9 & y^3-3y^2-8y-24 \\
 & \pm y^3 \pm 3y^2 \mp 3y \mp 9 \\ \hline
 & -5y-15 \\
 & -5(y+3)
\end{array}$$

By Ignoring -5

$$\begin{array}{r|c}
 & y^2-3 \\ \hline
y+3 & y^3+3y^2-3y-9 \\
 & \pm y^3 \pm 3y^2 \\ \hline
 & -3y-9 \\
 & \mp 3y \mp 9 \\ \hline
 & 0
\end{array}$$

So H.C.F is $y+3$

Q4. Find the L.C.M. of the following by factorization.

$12x^2-75, 6x^2-13x-5, 4x^2-20x+25$

Solution:

$$\begin{aligned}
12x^2-75 &= 3(4x^2-25) = 3[(2x)^2-5^2] \\
&= 3(2x-5)(2x+5) \\
6x^2-13x-5 &= 6x^2-15x+2x-5 \\
&= 3x(2x-5)(3x+1) \\
&= (2x-5)(3x+1) \\
4x^2-20x+25 &= (2x)^2-2(2x)(5)+25 \\
&= 3x(2x-5)(3x+1) \\
&= (2x-5)^2 \\
\text{So L.C.M.} &= \mathbf{3\,(2x+5)(3x+1)(2x-5)^2}
\end{aligned}$$

Q5. If H.C.F. of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7, Find their L.C.M.

Solution:

Let $p(x) = x^4+3x^3+5x^2+26x+56$

$q(x) = x^4+2x^3-4x^2-x+28$

$\text{H.C.F.} = x^2+5x+7$

$$\text{L.C.M.} = \frac{p(x)\times q(x)}{\text{H.C.F}}$$

$$= \frac{(x^4+3x^3+5x^2+26x+56).(x^4+2x^3-4x^2-x+28)}{x^2+5x+7}$$

Now dividing

$$\begin{array}{r|l} & x^2-2x+8 \\ \hline x^2+5x+7 & x^4+3x^3+5x^2+26x+56 \\ & \pm x^4 \pm 5x^3 \pm 7x^2 \\ \hline & -2x^3-2x^2+26x \\ & \mp 2x^3 \mp 10x^2 \mp 14x \\ \hline & 8x^2+40x+56 \\ & \pm 8x^2 \pm 40x \pm 56 \\ \hline & 0 \end{array}$$

So L.C.M.

$$= \frac{(x^2+5x+7)(x^2-2x+8)(x^4+2x^3-4x^2-x+28)}{x^2+5x+7}$$

$$= (x^2-2x+8)(x^4+2x^3-4x^2-x+28)$$

Q6. Simplify

(i) $\dfrac{3}{x^3+x^2+x+1} - \dfrac{3}{x^3-x^2+x-1}$

Solution:

$$= \frac{3}{x^2(x+1)+1(x+1)} - \frac{3}{x^2(x-1)+1(x-1)}$$

$$= \frac{3}{(x+1)(x^2+1)} - \frac{3}{(x-1)(x^2+1)}$$

$$= \frac{3(x-1)-3(x+1)}{(x+1)(x-1)(x^2+1)}$$

$$= \frac{3x-3-3x-3}{(x^2-1)(x^2+1)}$$

$$= \frac{-6}{x^4-1}$$

$$= \frac{-6}{-(1-x^4)}$$

$$= \frac{6}{1-x^4}$$

(ii) $\frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$

Solution:

$$= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab}$$

$$= \frac{a+b}{(a+b)(a-b)} \times \frac{(a-b)(a-b)}{a(a-b)} \qquad = \frac{1}{a}$$

Q7. Find square root by using factorization.

$$\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \qquad (x \neq 0)$$

Solution:

$$= x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+27-2$$

$$= \left(x+\frac{1}{x}\right)^2+10\left(x+\frac{1}{x}\right)+25$$

$$= \left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)+(5)+(5)^2$$

$$= \left[\left(x+\frac{1}{x}\right)+5\right]^2$$

Required square root is $\pm\left[\left(x+\frac{1}{x}\right)+5\right]$

Q8. Find square root by using division method.

$$\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \quad (x \neq 0)(y \neq 0)$$

Solution:

$$\begin{array}{l|l}
 & \frac{2x}{y}+5-\frac{3y}{x} \\
\hline
\frac{2x}{y} & \frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \\
 & \pm\frac{4x^2}{y^2} \\
\hline
\frac{4x}{y}+5 & \frac{20x}{y}+13 \\
 & \mp\frac{20x}{y}\pm 25 \\
\hline
\frac{4x}{y}+10-\frac{3y}{x} & -12-\frac{30y}{x}+\frac{9y^2}{x^2} \\
 & \pm 12 \mp \frac{30y}{x} \pm \frac{9y^2}{x^2} \\
\hline
 & 0
\end{array}$$

So the required root is $\pm\left(\frac{2x}{y}+5-\frac{3y}{x}\right)$